Non-reflective power extraction from a generator coil[1]

Andreas Höschler

Jun 20, 2013

[1]Created with Cassiopeia for MacOSX - See http://www.advanced-science.com

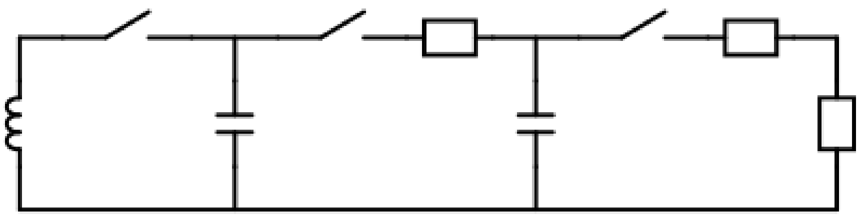
Abstract

Attaching a resistive load directly to a generator coil leads to lugging due to lenz law. Many ideas have been presented to extract energy from a generator coil without this lugging effect (non-reflecting to the power source). One is to attach a cap to the generator coil for the first quarter of the sine wave and then disconnect the cap from the coil before dumping its charge into a load. Another suggestion is to dump the charge of the cap into another cap or a battery (energy reservoir). This paper discusses the second approach and shows how to avoid/reduce the apparent heat loss during the energy transfer.

Contents

1 Energy transfer from one capacitor to another

It was suggested to extract power from a generator coil into a fist cap, to disconnect this cap from the generator coil and connect it to a second cap, then disconnect the second cap from the first and pump the charge of the second cap into a load.



The horizontal resistors represent the wire resistance, the vertical resistor at the right represents the load. One capacitor would be sufficient to separate a resistive load from the gen coil. However, one might want to connect a small HV cap to the gen coil for power extraction and have a bigger LV cap (or a battery) as an energy reservoir.

We assume a voltage U_0 in C_1 and no voltage in C_2 at $t = 0$. The switch between the two caps is closed allowing current to flow from the first to the second capacitor.

$$\begin{aligned}
U_1 &= RI + U_2 \\
U_0 - \frac{1}{C_1}\int I\ dt &= RI + \frac{1}{C_2}\int I\ dt \\
U_0 - RI &= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int I\ dt \\
Q &= \int I\ dt \\
U_0 - R\frac{dQ}{dt} &= \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q
\end{aligned}$$

$$R\frac{dQ}{dt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)Q=U_0 \tag{1}$$

We first solve the homogeneous part of this differential equation with the following approach:

$$\begin{aligned} Q(t) &= Q_0 e^{kt} \\ Q'(t) &= Q_0 k e^{kt} \end{aligned}$$

$$\begin{aligned} RQ_0 k e^{kt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)Q_0 e^{kt} &= 0 \\ \left(Rk+\frac{1}{C_1}+\frac{1}{C_2}\right)Q_0 e^{kt} &= 0 \end{aligned}$$

This is true for all t only if

$$Rk+\frac{1}{C_1}+\frac{1}{C_2}=0$$

$$\begin{aligned} k &= -\frac{\frac{1}{C_1}+\frac{1}{C_2}}{R} \\ k &= -\frac{1}{R}\frac{C_1+C_2}{C_1C_2} \end{aligned}$$

A solution for the homogeneous differential equation is therefore

$$Q(t) = Q_0 e^{kt} \tag{2}$$

$$k = -\frac{1}{R}\frac{C_1+C_2}{C_1C_2} \tag{3}$$

We find a solution of the inhomogeneous differential equation by varying the factor.

$$Q(t) = K(t)\,e^{kt} \tag{4}$$

$$Q'(t) = \frac{d(K(t))}{dt}e^{kt}+K(t)\,ke^{kt} \tag{5}$$

$$R\frac{dQ}{dt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)Q = U_0$$

$$
\begin{aligned}
R\left(\frac{d\left(K\left(t\right)\right)}{dt}e^{kt}+K\left(t\right)ke^{kt}\right)+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)K\left(t\right)e^{kt} &= U_0\\
R\left(\frac{d\left(K\left(t\right)\right)}{dt}e^{kt}+K\left(t\right)ke^{kt}\right)-kRK\left(t\right)e^{kt} &= U_0\\
R\frac{d\left(K\left(t\right)\right)}{dt}e^{kt}+RK\left(t\right)ke^{kt}-kRK\left(t\right)e^{kt} &= U_0\\
\left(\frac{d\left(K\left(t\right)\right)}{dt}+kK\left(t\right)-kK\left(t\right)\right)Re^{kt} &= U_0
\end{aligned}
$$

$$
\begin{aligned}
K\left(t\right) &= \frac{U_0}{R}\int e^{-kt}\ dt\\
K\left(t\right) &= \frac{U_0}{R}\int e^{-kt}\ dt\\
u &= -kt\\
\frac{du}{dt} &= -k
\end{aligned}
$$

$$
\begin{aligned}
K\left(t\right) &= -\frac{1}{k}\frac{U_0}{R}\int e^{u}\ du\\
K\left(t\right) &= -\frac{1}{k}\frac{U_0}{R}e^{u}+A
\end{aligned}
$$

We substitute this result into Eq. 4

$$
Q\left(t\right)=\left(-\frac{1}{k}\frac{U_0}{R}e^{-kt}+A\right)e^{kt}
$$

and thus get the general solution for the inhomogenous differential equation Eq. 1:

$$
Q\left(t\right)=Ae^{kt}-\frac{1}{k}\frac{U_0}{R} \tag{6}
$$

Since $U_2=Q/C_2$ we have

$$
U_2\left(t\right)=\frac{1}{C_2}\left(Ae^{kt}-\frac{1}{k}\frac{U_0}{R}\right)
$$

If we assume $U_2\left(0\right)=0$

$$
\begin{aligned}
0 &= A-\frac{1}{k}\frac{U_0}{R}\\
A &= \frac{1}{k}\frac{U_0}{R}
\end{aligned}
$$

we get

$$U_2(t) = \frac{1}{C_2}\left(\frac{1}{k}\frac{U_0}{R}e^{kt} - \frac{1}{k}\frac{U_0}{R}\right) \quad (7)$$

$$U_2(t) = \frac{1}{k}\frac{U_0}{RC_2}\left(e^{kt} - 1\right) \quad (8)$$

We substitute A into Eq. 6 and get

$$Q(t) = \frac{1}{k}\frac{U_0}{R}e^{kt} - \frac{1}{k}\frac{U_0}{R} \quad (9)$$

$$Q(t) = \frac{1}{k}\frac{U_0}{R}\left(e^{kt} - 1\right) \quad (10)$$

We get the current by taking the derivative of Eq. 10:

$$I(t) = \frac{d(Q(t))}{dt} \quad (11)$$

$$I(t) = \frac{1}{k}\frac{U_0}{R}ke^{kt} \quad (12)$$

$$I(t) = \frac{U_0}{R}e^{kt} \quad (13)$$

The voltage in the source cap is given by

$$U_1(t) = RI(t) + U_2(t) \quad (14)$$

$$U_1(t) = R\frac{U_0}{R}e^{kt} + \frac{1}{C_2}\frac{1}{k}\frac{U_0}{R}\left(e^{kt} - 1\right) \quad (15)$$

$$U_1(t) = U_0e^{kt} + \frac{1}{k}\frac{U_0}{RC_2}\left(e^{kt} - 1\right) \quad (16)$$

We plot these three functions.

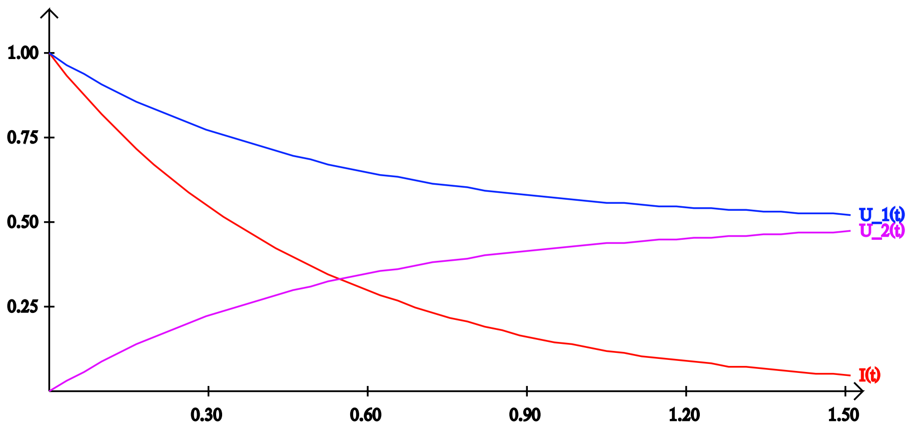


Figure 1: C_1=1; C_2=1; R=1; U_0=1

We have

$$
\begin{aligned}
U_1(\infty) &= -\frac{1}{k}\frac{U_0}{RC_2} \\
U_2(\infty) &= -\frac{1}{k}\frac{U_0}{RC_2} \\
I(\infty) &= 0
\end{aligned}
$$

with

$$
k = -\frac{1}{R}\frac{C_1 + C_2}{C_1 C_2}
$$

1.1 Energy Consideration

The initial energy in the system is given by

$$
E_{before} = \frac{1}{2} C_1 {U_0}^2
$$

The energy after the charge transfer is given by

$$
\begin{aligned}
E_{after} &= \frac{1}{2} C_1 (U_1(\infty))^2 + \frac{1}{2} C_2 (U_2(\infty))^2 \\
E_{after} &= \frac{1}{2} C_1 \left(-\frac{1}{k}\frac{U_0}{RC_2}\right)^2 + \frac{1}{2} C_2 \left(-\frac{1}{k}\frac{U_0}{RC_2}\right)^2 \\
E_{after} &= \frac{1}{2}\frac{1}{k^2}\left(\frac{U_0}{RC_2}\right)^2 (C_1 + C_2) \\
E_{after} &= \frac{1}{2}\frac{1}{\left(-\frac{1}{R}\frac{C_1+C_2}{C_1 C_2}\right)^2}\left(\frac{U_0}{RC_2}\right)^2 (C_1 + C_2) \\
E_{after} &= \frac{1}{2}\frac{R^2}{\left(\frac{C_1+C_2}{C_1 C_2}\right)^2}\left(\frac{U_0}{RC_2}\right)^2 (C_1 + C_2) \\
E_{after} &= \frac{1}{2}\frac{{C_1}^2 {C_2}^2}{(C_1 + C_2)^2}\left(\frac{U_0}{C_2}\right)^2 (C_1 + C_2) \\
E_{after} &= \frac{1}{2}\frac{{C_1}^2 {C_2}^2}{C_1 + C_2}\left(\frac{U_0}{C_2}\right)^2 \\
E_{after} &= \frac{1}{2}\frac{{C_1}^2}{C_1 + C_2}{U_0}^2 \\
E_{after} &= \frac{1}{2}\frac{C_1}{C_1 + C_2} C_1 {U_0}^2
\end{aligned}
$$

The difference between these two energies is the heat loss.

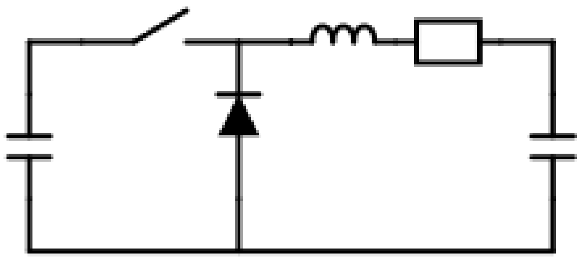
$$
\begin{aligned}
E_{heat} &= E_{before} - E_{after} \\
E_{heat} &= \frac{1}{2} C_1 {U_0}^2 - \frac{1}{2} \frac{C_1}{C_1 + C_2} C_1 {U_0}^2 \\
E_{heat} &= \frac{1}{2} C_1 {U_0}^2 \left(1 - \frac{C_1}{C_1 + C_2} \right) \\
E_{heat} &= E_{before} \left(1 - \frac{C_1}{C_1 + C_2} \right)
\end{aligned}
$$

$$
\kappa = \frac{E_{heat}}{E_{heat}} = 1 - \frac{C_1}{C_1 + C_2} \tag{17}
$$

The goal was to pump energy from a small HV cap into a larger LV cap (energy reservoir). This means we have $C_1 \ll C_2$ and thus $\kappa \approx 1$. Almost the entire energy is lost in the wire resistance as heat. For $C_1 = C_2$ we have $\kappa = 0.5$ meaning that half the energy is lost. Note that κ is independent of R so using thick wire wouldn't help a bit.

2 Energy transfer from one capacitor to another via a choke

We have shown in **1 Energy transfer from one capacitor to another** that transfering energy from one cap to another by means of a wire with R>0 causes an unacceptable heat loss. For two identical caps half of the energy is lost. For a smaller source cap the outcome is even worse. It is therefore suggested to introduce a choke into the circuit as shown below.



Once the source cap has reached a low enough potential the coil becomes an active component and converts its magnetic energy back into current and thus contributes to the filling of the target cap. The above circuit (switch closed) is described by the following differential equation.

$$
U_1 = L \frac{dI}{dt} + RI + U_2
$$

$$U_0 - \frac{Q}{C_1} = L\frac{dI}{dt} + RI + \frac{Q}{C_2}$$

$$\begin{aligned} U_0 &= L\frac{dI}{dt} + RI + \frac{Q}{C_2} + \frac{Q}{C_1} \\ U_0 &= LQ'' + RQ' + Q\frac{C_1 + C_2}{C_1 C_2} \end{aligned}$$

$$Q'' + \frac{R}{L}Q' + Q\frac{1}{L}\frac{C_1 + C_2}{C_1 C_2} = \frac{U_0}{L} \tag{18}$$

We solve the homogeneous differential equation

$$Q'' + \frac{R}{L}Q' + Q\frac{1}{L}\frac{C_1 + C_2}{C_1 C_2} = 0 \tag{19}$$

first using the following approach:

$$\begin{aligned} Q &= Q_0 e^{i\omega t} \\ Q' &= Q_0 i\omega e^{i\omega t} \\ Q'' &= -Q_0 \omega^2 e^{i\omega t} \end{aligned}$$

$$\begin{aligned} -Q_0\omega^2 e^{i\omega t} + \frac{R}{L}Q_0 i\omega e^{i\omega t} + Q_0 e^{i\omega t}\frac{1}{L}\frac{C_1 + C_2}{C_1 C_2} &= 0 \\ \left(-\omega^2 + \frac{R}{L}i\omega + \frac{1}{L}\frac{C_1 + C_2}{C_1 C_2}\right) Q_0 e^{i\omega t} &= 0 \end{aligned}$$

This equation is true for all t if we have

$$\begin{aligned} -\omega^2 + \frac{R}{L}i\omega + \frac{1}{L}\frac{C_1 + C_2}{C_1 C_2} &= 0 \\ \omega^2 - \frac{R}{L}i\omega - \frac{1}{L}\frac{C_1 + C_2}{C_1 C_2} &= 0 \end{aligned}$$

This equation is solved by

$$\begin{aligned} \omega_1 &= -\frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q} \\ \omega_2 &= -\frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 - q} \end{aligned}$$

with

$$
\begin{aligned}
p &= -\frac{R}{L}i \\
q &= -\frac{1}{L}\frac{C_1+C_2}{C_1C_2}
\end{aligned}
$$

We consider ω_1 first.

$$
\begin{aligned}
\omega_1 &= -\frac{-\frac{R}{L}}{2}i + \sqrt{-\left(\frac{-\frac{R}{L}}{2}\right)^2 - \left(-\frac{1}{L}\frac{C_1+C_2}{C_1C_2}\right)} \\
\omega_1 &= \frac{R}{2L}i + \sqrt{-\left(\frac{R}{2L}\right)^2 + \frac{1}{L}\frac{C_1+C_2}{C_1C_2}}
\end{aligned}
$$

$$
\begin{aligned}
Q &= Q_0 e^{i\omega_1 t} \\
Q &= Q_0 e^{i\left(\frac{R}{2L}i + \sqrt{-\left(\frac{R}{2L}\right)^2 + \frac{1}{L}\frac{C_1+C_2}{C_1C_2}}\right)t} \\
Q &= Q_0 e^{\left(-\frac{R}{2L} + i\sqrt{-\left(\frac{R}{2L}\right)^2 + \frac{1}{L}\frac{C_1+C_2}{C_1C_2}}\right)t} \\
Q &= Q_0 e^{i\sqrt{\frac{1}{L}\frac{C_1+C_2}{C_1C_2} - \left(\frac{R}{2L}\right)^2}t} e^{\left(-\frac{R}{2L}\right)t}
\end{aligned}
$$

We can rewrite this as

$$
Q = \left(A\cos\left(\eta t\right) + B\sin\left(\eta t\right)\right) e^{\left(-\frac{R}{2L}\right)t} \tag{20}
$$

$$
\eta = \sqrt{\frac{1}{L}\frac{C_1+C_2}{C_1C_2} - \left(\frac{R}{2L}\right)^2} \tag{21}
$$

We solve the inhomogeneous differential equation by finding a particular solution $K\left(t\right)$ as follows:

$$
\begin{aligned}
K\left(t\right) &= K_0 \\
K'\left(t\right) &= 0 \\
K''\left(t\right) &= 0
\end{aligned}
$$

We substitute this as Q into Eq. 18.

$$
Q'' + \frac{R}{L}Q' + Q\frac{1}{L}\frac{C_1+C_2}{C_1C_2} = \frac{U_0}{L}
$$

$$
\begin{aligned}
K_0 \frac{1}{L} \frac{C_1 + C_2}{C_1 C_2} &= \frac{U_0}{L} \\
K_0 &= U_0 \frac{C_1 C_2}{C_1 + C_2}
\end{aligned}
$$

The general solution for Eq. 18 using ω_1 is then

$$
Q(t) = \left(A \cos(\eta t) + B \sin(\eta t)\right) e^{\left(-\frac{R}{2L}\right) t} + U_0 \frac{C_1 C_2}{C_1 + C_2}
$$

We demand $Q(0) = 0$. This gets us

$$
\begin{aligned}
0 &= A + U_0 \frac{C_1 C_2}{C_1 + C_2} \\
A &= -U_0 \frac{C_1 C_2}{C_1 + C_2}
\end{aligned}
$$

We further demand $I(0) = 0:$

$$
Q(t) = \left(A \cos(\eta t) + B \sin(\eta t)\right) e^{\left(-\frac{R}{2L}\right) t} + U_0 \frac{C_1 C_2}{C_1 + C_2}
$$

$$
\begin{aligned}
Q'(t) &= \left(-A\eta \sin(\eta t) + B\eta \cos(\eta t)\right) e^{-\frac{R}{2L} t} - \left(A \cos(\eta t) + B \sin(\eta t)\right) \frac{R}{2L} e^{-\frac{R}{2L} t} \\
0 &= B\eta - A \frac{R}{2L} \\
B\eta &= A \frac{R}{2L} \\
B &= \frac{1}{\eta} \left(-U_0 \frac{C_1 C_2}{C_1 + C_2}\right) \frac{R}{2L}
\end{aligned}
$$

The special solution for our problem honoring our start conditions is therefore

$$
\begin{aligned}
Q(t) &= \left(\left(-U_0 \frac{C_1 C_2}{C_1 + C_2}\right) \cos(\eta t) + \frac{1}{\eta} \left(-U_0 \frac{C_1 C_2}{C_1 + C_2}\right) \frac{R}{2L} \sin(\eta t)\right) e^{\left(-\frac{R}{2L}\right) t} + U_0 \frac{C_1 C_2}{C_1 + C_2} \\
Q(t) &= -\frac{C_1 C_2}{C_1 + C_2} \left(U_0 \cos(\eta t) + \frac{1}{\eta} U_0 \frac{R}{2L} \sin(\eta t)\right) e^{-\frac{R}{2L} t} + U_0 \frac{C_1 C_2}{C_1 + C_2} \\
Q(t) &= -U_0 \frac{C_1 C_2}{C_1 + C_2} \left(\cos(\eta t) + \frac{1}{\eta} \frac{R}{2L} \sin(\eta t)\right) e^{-\frac{R}{2L} t} + U_0 \frac{C_1 C_2}{C_1 + C_2}
\end{aligned}
$$

$$
Q(t) = U_0 \frac{C_1 C_2}{C_1 + C_2} \left(1 - \left(\cos(\eta t) + \frac{1}{\eta} \frac{R}{2L} \sin(\eta t)\right) e^{-\frac{R}{2L} t}\right) \tag{22}
$$

The voltage in the target cap is given by

$$
\begin{aligned}
U_2(t) &= \frac{Q(t)}{C_2} \\
U_2(t) &= U_0 \frac{C_1}{C_1 + C_2} \left(1 - \left(\cos(\eta t) + \frac{1}{\eta} \frac{R}{2L} \sin(\eta t) \right) e^{-\frac{R}{2L} t} \right)
\end{aligned}
$$

The current is given by

$$
I(t) = \frac{dQ}{dt}
$$

$$
\begin{aligned}
I(t) &= -U_0 \frac{C_1 C_2}{C_1 + C_2} \left(\left(-\eta \sin(\eta t) + \frac{1}{\eta} \frac{R}{2L} \eta \cos(\eta t) \right) e^{-\frac{R}{2L} t} - \left(\cos(\eta t) + \frac{1}{\eta} \frac{R}{2L} \sin(\eta t) \right) \frac{R}{2L} e^{-\frac{R}{2L} t} \right) \\
I(t) &= -U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(-\eta \sin(\eta t) + \frac{R}{2L} \cos(\eta t) - \left(\cos(\eta t) + \frac{1}{\eta} \frac{R}{2L} \sin(\eta t) \right) \frac{R}{2L} \right) \\
I(t) &= -U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(-\eta \sin(\eta t) + \frac{R}{2L} \cos(\eta t) - \left(\frac{R}{2L} \cos(\eta t) + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \sin(\eta t) \right) \right) \\
I(t) &= -U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(-\eta \sin(\eta t) + \frac{R}{2L} \cos(\eta t) - \frac{R}{2L} \cos(\eta t) - \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \sin(\eta t) \right) \\
I(t) &= U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(\eta \sin(\eta t) + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \sin(\eta t) \right) \\
I(t) &= U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(\eta + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \right) \sin(\eta t)
\end{aligned}
$$

The voltage in the source cap is given by

$$
U_1(t) = L I'(t) + R I(t) + U_2(t) \tag{23}
$$

with

$$
\begin{aligned}
I'(t) &= U_0 \frac{C_1 C_2}{C_1 + C_2} \left(-\frac{R}{2L} e^{-\frac{R}{2L} t} \left(\eta + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \right) \sin(\eta t) + e^{-\frac{R}{2L} t} \left(\eta + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \right) \eta \cos(\eta t) \right) \\
I'(t) &= U_0 \frac{C_1 C_2}{C_1 + C_2} e^{-\frac{R}{2L} t} \left(\eta + \frac{1}{\eta} \left(\frac{R}{2L} \right)^2 \right) \left(-\frac{R}{2L} \sin(\eta t) + \eta \cos(\eta t) \right)
\end{aligned}
$$

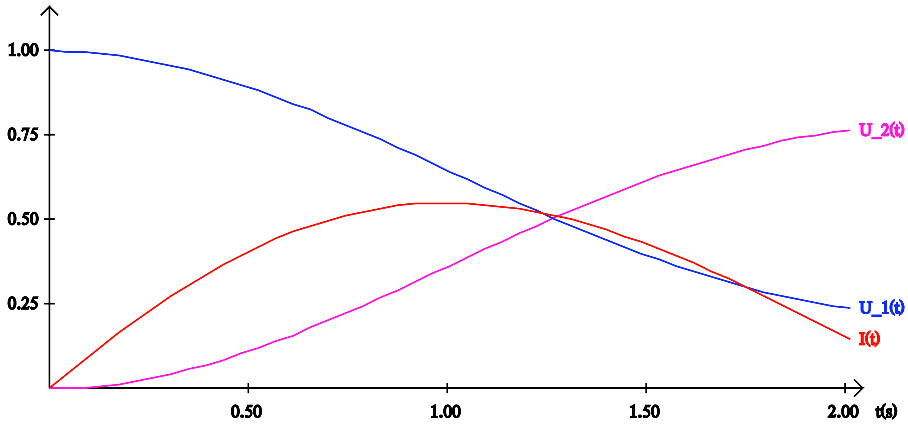


Figure 2: C_1=1; C_2=1; L=1; R=0.5; U_0=1

The different outcome is obvious and this time highly dependent on the size of the wire resistance R.

2.1 Energy consideration

The energy in C_1 at $t = 0$ is given by

$$E_{before}(t) = \frac{1}{2} C_1 {U_0}^2$$

The energy in target cap after time t is given by

$$E_{after}(t) = \frac{1}{2} C_2 (U_2(t))^2$$

The following figure shows the development of E_{after} (energy in C_2) in relation to the initial input energy E_{before}.

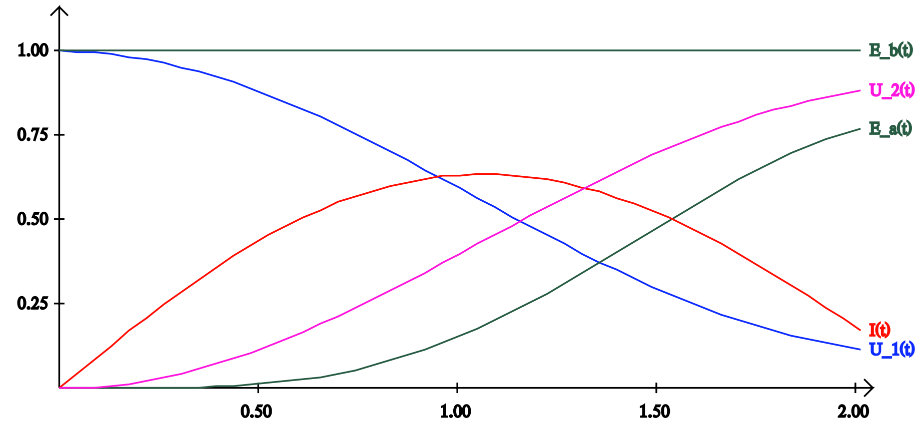


Figure 3: C_1=1; C_2=1; L=1; R=0.2; U_0=1; Factor E_b(t): 2.00; Factor E_a(t): 2.00

There is a point with maximal current in the choke. The voltage potential between C_1 and C_2 plus the voltage drop over R has dropped to zero and the choke starts to discharge its magentic energy into the target cap. The heat loss is minimal inspite of a significant wire resistance in this example. We create a new plot with more likely values for the components.

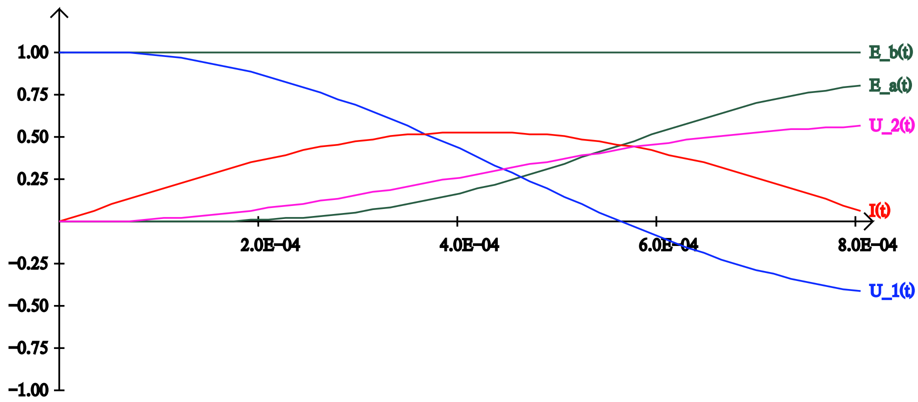


Figure 4: C_1=2E-6; C_2=5E-6; L=50E-3; R=0.2; U_0=1; Factor E_a(t): 1.0E+06; Factor E_b(t): 1.0E+06; Factor I(t): 100.00

Our circuit equations do not honor the diode that suppresses oscillations and prevents C_1 from being charged negatively. The energy of the coil would mainly go into C_2 where it belongs with the diode in place. We can conclude that the choke approach transfers almost 100% of the energy in C_1 to C_2 with no apparent heat loss. In a real application the inductivity should be chosen as small as possible to ensure fast enough energy transfer from left to right but large enough to prevent significant voltage drops over the wire.

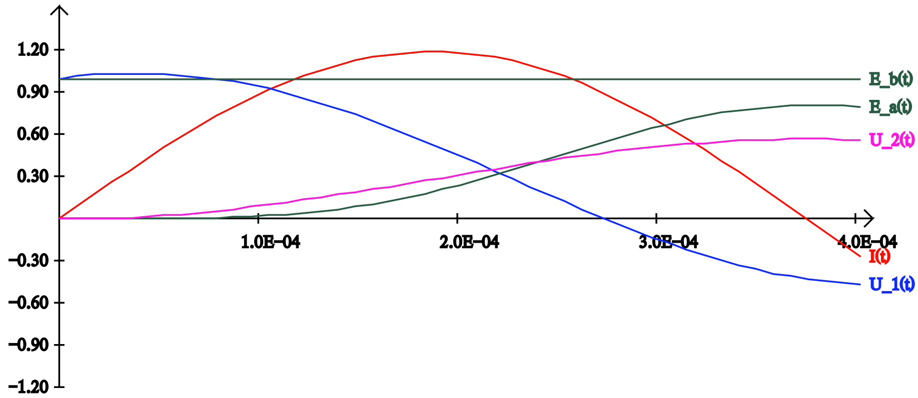


Figure 5: C_1=2E-6; C_2=5E-6; L=10E-3; R=0.2; U_0=1; Factor I(t): 100.00; Factor E_b(t): 1.0E+06; Factor E_a(t): 1.0E+06

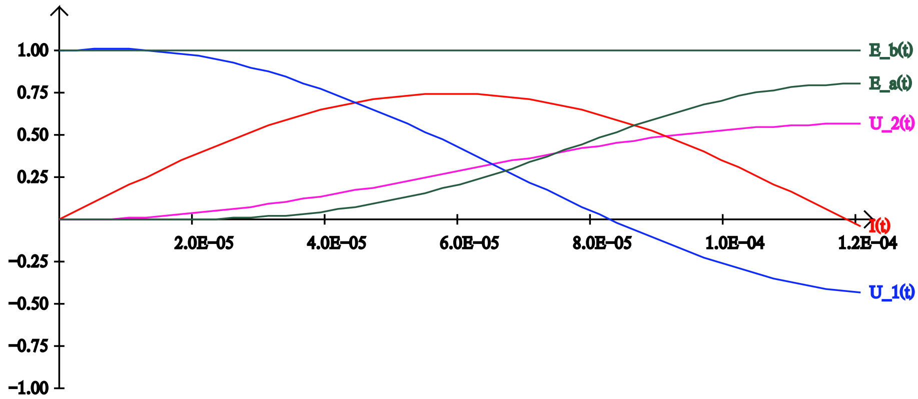


Figure 6: C_1=2E-6; C_2=5E-6; L=1E-3; R=0.2; U_0=1; Factor I(t): 20.00; Factor E_b(t): 1.0E+06; Factor E_a(t): 1.0E+06

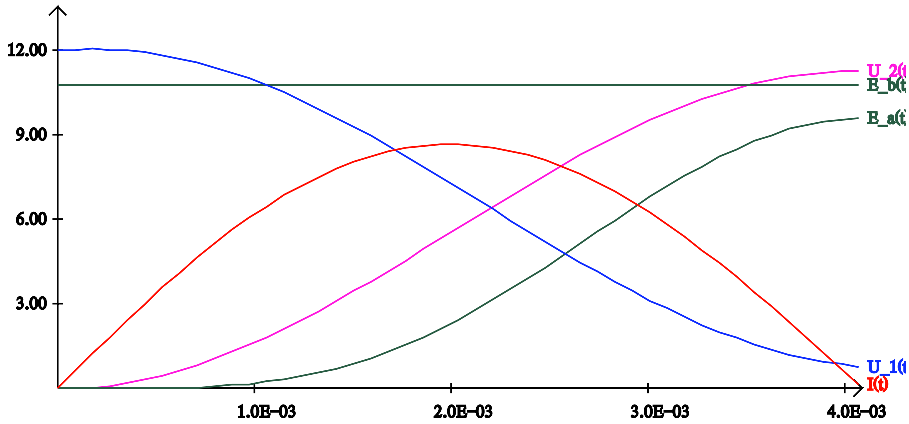


Figure 7: C_1=1000E-6; C_2=1000E-6; L=3.4E-3; R=0.2; U_0=12; Factor E_a(t): 150.00; Factor E_b(t): 150.00; Factor I(t): 2.00